Basic principles of switch-mode power conversion (DC-DC): Cycle-by-cycle averaging

- Bi-positional switch
- PWM and duty ratio
- Cycle-by-cycle averaging
  - Concept of DC steady-state balance
  - Current-second balance
  - Power balance

Step-down converter example

- By controlling the ON/OFF durations of the switches, we control the periodic average of bi-positional switch output, and in turn the average values of other quantities
- Most quantities have switching waveforms or high-frequency content, but for most control purposes we are only interested in average value

Definition of cycle-by-cycle averaging

- Average over a switching period
- CCA values denoted by a bar ( - ) on top, like \( q_A \)

CCA examples: DC-DC converters

CCA examples: sinusoidal PWM

CCA calculation and implementation in simulation

\[
\tilde{v}_d(t) = \frac{1}{T_s} \int_{t-T_s}^{t} v_d(t) \, dt
\]

The term \( \frac{1}{T_s} \int_{t-T_s}^{t} v_d(t) \, dt \) can be implemented by

either delaying the integral \( \int_0^T v_d(t) \, dt \) by \( T_s \) or as \( \int_0^T v_d(t) \, dt - \int_0^{T_s} v_d(t) \, dt \) to result in

\[
\tilde{v}_d(t) = v_d(t-T_s) - \frac{T_s}{T} \int_0^{T_s} v_d(t) \, dt
\]

Cycle-by-cycle averaging (CCA)

- Average over a switching period referred to as cycle-by-cycle average (CCA)
- Control objectives achieved essentially by controlling the CCA value of different quantities
- Average models, steady-state analysis & controller design use CCA quantities

CCA examples: DC-DC converters

CCA examples: sinusoidal PWM
Some properties of CCA

- Just like instantaneous quantities, KCL and KVL apply for CCA quantities too.

KCL
\[ \sum_i x_i = 0 \]
At a node

KVL
\[ \sum_i V_x = 0 \]
Around a loop

\[ \dot{x}_1 + \dot{x}_2 + \dot{x}_3 = 0 \]
Integrating both sides and dividing by \( T_c \)

\[ \frac{1}{T_c} \int_0^T x_1 dt + \frac{1}{T_c} \int_0^T x_2 dt + \frac{1}{T_c} \int_0^T x_3 dt = 0 \]

CCA at node \( x \)

CCA KCL and KVL example

CCA KVL around loop \( y \)

Some properties of CCA

- V-I relationship in CCA for R, L and C similar to the instantaneous relationships

Instantaneous CCA

\[ \tau(t) = R i(t) \]

\[ v_1(t) = L \frac{di(t)}{dt} \]

\[ v_2(t) = \frac{1}{C} \int v_3(t) dt \]

\[ \tau_1(t) = \frac{1}{L} \int i(t) dt \]

\[ \tau_2(t) = \frac{1}{C} \int v(t) dt \]

- The derivative is given by

\[ \frac{d}{dt} \tau(t) = \frac{1}{T_c} \left[ s(t) - s(t - T_c) \right] \]

Applications

- CCA can be used in both steady-state and transient analysis

- Simulations based on CCA models are sometimes orders of magnitude faster

- Since the process of CCA removes the switching frequency component and its harmonics, phasor analysis can be applied (at fundamental frequency) in sinusoidal applications

- CCA analysis cannot be used for studying switching frequency ripple, switch stress and other high frequency effects

Some properties of CCA: Derivative

\[ \tau(t) = \frac{1}{T_c} \int s(t) dt \]
(CCA definition)

Differentiating both sides,

\[ \frac{d}{dt} \tau(t) = \frac{d}{dt} \left[ \frac{1}{T_c} \int s(t) dt \right] \]

\[ \frac{d}{dt} \tau(t) = \frac{1}{T_c} s(t) \]

(From second fundamental theorem of calculus)